

B.Sc. II Semester Degree Examination, April/May. - 2019

MATHEMATICS

Calculus-I

(New)

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates:

Part A : All questions are Compulsory.

Part B: Solve any **Five** from Seven questions.**PART - A****L** Answer **ALL** the following.

(10×2=20)

1. State Rolle's theorem and show that it is inapplicable for $f(x) = x^3$ in $[1, 2]$.2. Verify Lagrange's mean value theorem for $f(x) = (x^2 - 4)^{\frac{1}{2}}$ in $[2, 3]$.3. Expand $f(x) = e^{-x}$ up to the terms containing x^5 using Maclaurins Series.4. Evaluate $\lim_{x \rightarrow 0} \frac{xe^x - \log(1+x)}{x^2}$ 5. If $y = x^m$, where m is a positive integer then show that $y_n = \frac{m!}{(m-n)!} x^{m-n}$. Also discuss the possibilities when $n = m$ and $n > m$.6. Find the n^{th} derivative of $y = \sin^3 2x$ 7. If $y = e^{m \cos^{-1} x}$ prove that $(1-x^2)y_2 - xy_1 = m^2 y$ 8. If $x = r \cos \theta$ and $y = r \sin \theta$ then $\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 = 1$

[P.T.]



9. Find the total derivative of $u = x^2 - y^2$ where $x = e^t \cos t$, and $y = e^t \sin t$.

10. If $u = x(1+y) & v = y(1+x)$ show that $\frac{\partial(u,v)}{\partial(x,y)} = 1+x+y$.

PART - B

Answer any FIVE of the following.

(5×12=60)

II. 11. State and prove Cauchy's mean value theorem.

12. Obtain the Maclaurin's expansion of $\log(1+x)$ and hence deduce

$$\log \sqrt{\frac{1+x}{1-x}} = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots$$

III. 13. Evaluate a) $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$

b) $\lim_{x \rightarrow \infty} (a^{1/x} - 1)x$

14. Evaluate a) $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{1/x}$

b) $\lim_{x \rightarrow 0} \left(\frac{1}{x} \right)^{2 \sin x}$

IV. 15. Find the n^{th} derivative of $y = e^{ax} \cos(bx+c)$

16. If $y = \frac{x^4}{x^2 - 3x + 2}$ then find y_n .

V. 17. State and prove Leibnitz theorem on derivative of product of two functions.

18. Find the n^{th} derivative of $\cos^2 x \cdot \sin^4 x$



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VI. 19. If $y = e^{a \cos^{-1} x}$ show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$

20. State and prove Euler's theorem on homogeneous functions.

VII. 21. Verify Euler's theorem for $u = \sin(\sqrt{x}) + \cot^{-1}(\sqrt{y})$

22. If $u = \tan^{-1}(\sqrt{y})$ where $x^2 + y^2 = 4$. Show that $\frac{du}{dx} = \frac{1}{\sqrt{4 - x^2}}$.

VIII.23. If u, v are functions of r, s and r, s are functions of x, y then $\frac{\partial(u, v)}{\partial(r, s)} \frac{\partial(r, s)}{\partial(x, y)} = \frac{\partial(u, v)}{\partial(x, y)}$

24. If $u = \frac{x_2x_3}{x_1}$, $v = \frac{x_3x_1}{x_2}$, & $w = \frac{x_1x_2}{x_3}$ show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 4$.



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B.Sc. II Semester Degree Examination, April/May - 2019

MATHEMATICS

Calculus-II

Paper - 2.2

(Old)

Time : 3 Hours

Maximum Marks : 60

Instructions to Candidates:

1. Answer all the sections.
2. Mention the question numbers correctly.

SECTION-A

L Answer any TEN of the following questions. **(10×2=20)**

1. Show that $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

2. Integrate $\frac{\sec^2 x}{\tan^2 x - 4}$ w.r.t. 'x'

3. Evaluate $\int \frac{dx}{5 + 4 \cos x}$

4. Integrate $\frac{2x-3}{\sqrt{2x^2-7x+5}}$ w.r.t.x

5. Evaluate $\int \frac{xe^x}{(1+x)^2} dx$

6. Evaluate $\int x \sin 3x dx$

7. Evaluate $\int \frac{x-1}{(x-2)(x-3)} dx$

8. Evaluate $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\cos^{3/2} x + \sin^{3/2} x} dx$

[P.T.O.]

9. Evaluate $\int_0^1 e^x dx$ as limit of a sum

10. Evaluate $\int_0^{\pi/2} \cos^6 x dx$

11. If $u = x \tan y + y \tan x$ verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

12. If $x = r \cos \theta, y = r \sin \theta, z = z$ find $\frac{\partial(x, y, z)}{\partial(r, \theta, z)}$

SECTION-B

II. Answer any FIVE of the following questions.

(5×5=25)

13. Evaluate $\int \frac{dx}{(x-1)^2(x+1)}$

14. Evaluate $\int \frac{dx}{5 + 7 \cos x + \sin x}$

15. Evaluate $\int x^2 \tan^{-1} x dx$

16. Show that $\int_0^\pi \frac{x}{1+\sin x} dx = \pi$

17. Evaluate $\int_1^3 (3x^2 + 1) dx$ as a limit of a sum.

18. Find the reduction formula for $\int \sin^n x dx$. Where n is the integer hence evaluate

$$\int_0^{\pi/2} \sin^n x dx$$

19. Find the area of the Curve Asteroid $x^{2/3} + y^{2/3} = a^{2/3}$.

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SECTION-C

III. Answer any **THREE** of the following questions.

(3×5=15)

20. If $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ S.T $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

21. If $u = \sin^{-1} \left[\frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}} \right]$, show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144} (13 + \tan^2 u)$

22. If $x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta$ show that $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin \theta$

23. If u and v are functions of two independent variables s and t . And s and t themselves are

functions of two independent variables x and y then $\frac{\partial(u, v)}{\partial(s, t)} \times \frac{\partial(s, t)}{\partial(x, y)} = \frac{\partial(u, v)}{\partial(x, y)}$

24. If $x = r \cos \theta, y = r \sin \theta$, calculate $J = \frac{\partial(x, y)}{\partial(r, \theta)}$ and $J' = \frac{\partial(r, \theta)}{\partial(x, y)}$ also verify $J \cdot J' = 1$.

B.Sc. II Semester Degree Examination, April/May - 2019
MATHEMATICS

Algebra-II
Paper - 2.1
(Old)

Time : 3 Hours

Maximum Marks : 60

Instructions to Candidates:

1. Answer the questions section wise.
2. Write the question numbers correctly.

SECTION-A

I. Answer any **TEN** of the following questions. **(10×2=20)**

1. State Euclid's algorithm theorem.
2. Using Descartes rules of Signs find the number of positive and negative roots of $2x^7 - x^4 + 4x^3 - 5 = 0$
3. Diminish the roots of the equation $2x^5 - x^3 + 10x - 8 = 0$ by 5.
4. Increase the roots of the equation $x^4 - 24x^2 - 13x + 35 = 0$ by 2.
5. Define convergence and divergence of sequences.
6. Show that sequence $\left\{1 - \frac{1}{n}\right\}$ is a monotonic increasing sequence.
7. Test the convergence of the sequence whose n^{th} term is given $1 + (-1)^{2n}$.
8. Find the limit superior and limit inferior for the sequence $\{x_n\} = \{(-1)^n\}$
9. Test the series $1^3 + 2^3 + 3^3 + \dots + n^3$ for divergence.
10. Test the convergence of the series $\sum \sin\left(\frac{1}{n}\right)$
11. State D' Alembert's Ratio test.
12. State the Leibnitz theorem for an alternating series.

SECTION-B

II. Answer any **TWO** of the following questions. **(2×5=10)**

13. Solve the equation $x^3 - 18x - 35 = 0$ by cordon's method.

[P.T.O.]

14. Show that the equation $2x^7 + 3x^4 + 3x + k = 0$ has at least four imaginary roots for all values of k .
15. Solve $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$ by removing the second term.
16. Solve the equation $4x^4 - 24x^3 - 31x^2 + 6x - 8 = 0$ given that the sum of the two roots of the equation is zero.

SECTION-C

- III.** Answer any THREE of the following questions. $(3 \times 5 = 15)$

17. Prove that $\lim_{n \rightarrow \infty} (x_n \cdot y_n) = l \cdot m$ where $\{x_n\}$ and $\{y_n\}$ are sequences converging to l, m respectively.
18. Prove that a monotonic increasing sequence which is not bounded above diverges to $+\infty$.
19. Show that the sequence $\{x_n\}$ where $x_1 = 1$ and $x_n = \sqrt{2 + x_{n-1}}$ is convergent and converges to 2.
20. Find the limit of following sequence 0.7, 0.77, 0.777,.....
21. Show that the sequence $\{x_n\}$ defined by $x_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!}$ is convergent and $2 \leq \text{Limit } x_n \leq 3$.

SECTION-D

- IV.** Answer any THREE of the following questions. $(3 \times 5 = 15)$

22. State and prove P-series.
23. State and prove Cauchy's n^{th} root test.
24. Discuss the convergence of the series

a) $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$

b) $\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots$

25. Discuss the convergence $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \dots + \frac{x^n}{n^2 + 1}$

26. Test the convergence of $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \dots$